Introduction to grayscale image processing by mathematical morphology

G. Bertrand, J. Cousty, M. Couprie and **L. Najman**

MorphoGraph

2022

Outline of the lecture

1. Grayscale images

2. Operators on grayscale images

Images

Definition

- *Let* $\mathbb{V}$ *be a set of values*
- *An* image (on E with values in $\mathbb{V}$) *is a map* I *from* E *into* $\mathbb{V}$
- $I(x)$ *is called the* value *of the point (pixel)* x *for* I

Images

Definition

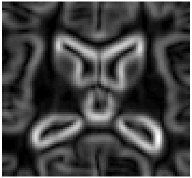
- *Let $\mathbb{V}$ be a set of values*
- *An image (on E with values in $\mathbb{V}$) is a map I from E into $\mathbb{V}$*
- *$I(x)$ is called the value of the point (pixel) x for I*

Example

- **Images with values in $\mathbb{R}^+$:** *euclidean distance map D_X to a set $X \in \mathcal{P}(E)$*
- **Images with values in $\mathbb{Z}^+$:** *distance map D_X for a geodesic distance in a uniform network*

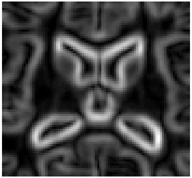
Grayscale images

- We denote by $\mathcal{I}$ the set of all images with integers values on E
- An image in $\mathcal{I}$ is also called *grayscale (or graylevel) image*



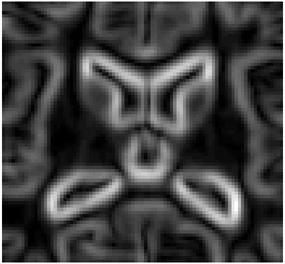
Grayscale images

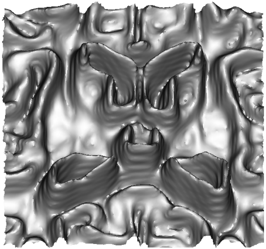
- We denote by $\mathcal{I}$ the set of all images with integers values on E
- An image in $\mathcal{I}$ is also called *grayscale (or graylevel) image*
- We denote by I an arbitrary image in $\mathcal{I}$
- The value $I(x)$ of a point $x \in E$ is also called the *gray level of x*, or the *gray intensity at x*



Topographical interpretation

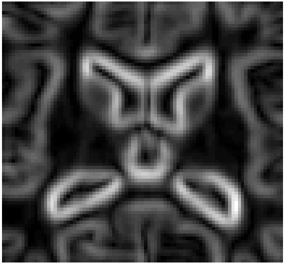
- An grayscale image I can be seen as a topographical relief
  - $I(x)$ is called the *altitude of x*

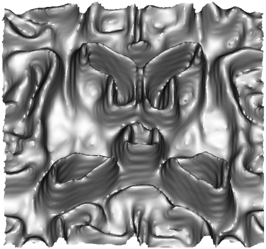




Topographical interpretation

- An grayscale image I can be seen as a topographical relief
  - $I(x)$ is called the *altitude of x*
  - Bright regions: mountains, crests, hills
  - Dark regions: bassins, valleys





Level set

Definition

- *Let* $k \in \mathbb{Z}$
- *The* k-level set (or k-section, or k-threshold) of I, *denoted by* I_k, *is the subset of* E *defined by:*
  - $I_k = \{x \in E \mid I(x) \geq k\}$

Level set

Definition

- *Let* $k \in \mathbb{Z}$
- *The* k-level set (or k-section, or k-threshold) of I, *denoted by* I_k, *is the subset of* E *defined by:*
  - $I_k = \{x \in E \mid I(x) \geq k\}$

I

I_{80}

I_{150}

I_{220}

Reconstruction

Property

- $\forall k, k' \in \mathbb{Z},\ k' > k \implies I_{k'} \subseteq I_k$
- $I(x) = \max\{k \in \mathbb{Z} \mid x \in I_k\}$

grayscale operators

- An *operator (on $\mathcal{I}$)* is a map from $\mathcal{I}$ into $\mathcal{I}$

grayscale operators

- An *operator (on $\mathcal{I}$)* is a map from $\mathcal{I}$ into $\mathcal{I}$

Definition (flat operators)

- *Let γ be an increasing operator on E*
- *The* stack operator induced by γ *is the operator on $\mathcal{I}$, also denoted by γ, defined by:*
  - $\forall I \in \mathcal{I}, \forall k \in \mathbb{Z}, [\gamma(I)]_k = \gamma(I_k)$

grayscale operators

- An *operator (on $\mathcal{I}$)* is a map from $\mathcal{I}$ into $\mathcal{I}$

Definition (flat operators)

- *Let γ be an increasing operator on E*
- *The* stack operator induced by γ *is the operator on $\mathcal{I}$, also denoted by γ, defined by:*
  - $\forall I \in \mathcal{I}, \forall k \in \mathbb{Z}, [\gamma(I)]_k = \gamma(I_k)$

Exercice. Show that a same construction cannot be used to derive an operator on $\mathcal{I}$ from an operator on E that is not increasing.

Characterisation of grayscale operators

Property

- *Let γ be an increasing operator on E*
- $[\gamma(I)](x) = \max\{k \in \mathbb{Z} \mid x \in \gamma(I_k)\}$

Characterisation of grayscale operators

Property

- *Let γ be an increasing operator on E*
- $[\gamma(I)](x) = \max\{k \in \mathbb{Z} \mid x \in \gamma(I_k)\}$

Remark. Untill now, all operators seen in the MorphoGraph and Imagery course are increasing

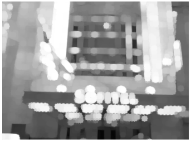
Illustration: dilation on $\mathcal{I}$ by Γ

I

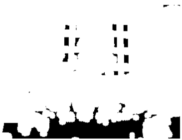
I_{80}

I_{150}

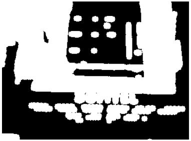
I_{220}



$\delta_\Gamma(I)$



$\delta_\Gamma(I)_{80}$



$\delta_\Gamma(I)_{150}$

$\delta_\Gamma(I)_{220}$

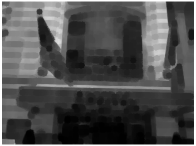
Illustration: erosion on $\mathcal{I}$ by Γ

I

I_{80}

I_{150}

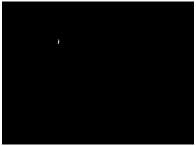
I_{220}



$\epsilon_{\Gamma}(I)$

$\epsilon_{\Gamma}(I)_{80}$

$\epsilon_{\Gamma}(I)_{150}$



$\epsilon_{\Gamma}(I)_{220}$

Illustration: opening on $\mathcal{I}$ by Γ

I

I_{80}

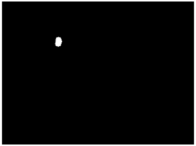
I_{150}

I_{220}

$\gamma_\Gamma(I)$

$\gamma_\Gamma(I)_{80}$

$\gamma_\Gamma(I)_{150}$



$\gamma_\Gamma(I)_{220}$

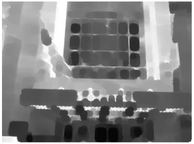
Illustration: closing on $\mathcal{I}$ by Γ

I

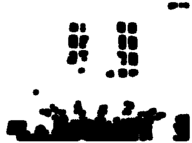
I_{80}

I_{150}

I_{220}



$\phi_\Gamma(I)$



$\phi_\Gamma(I)_{80}$

$\phi_\Gamma(I)_{150}$

$\phi_\Gamma(I)_{220}$

Dilation/Erosion by a structuring element: characterisation

Property (duality)

- *Let* Γ *be a structuring element*
- $\epsilon_{\Gamma}(I) = -\delta_{\Gamma^{-1}}(-I)$

Dilation/Erosion by a structuring element: characterisation

Property (duality)

- *Let* Γ *be a structuring element*
- $\epsilon_\Gamma(I) = -\delta_{\Gamma^{-1}}(-I)$

Property

- *Let* Γ *be a structuring element*
- $[\delta_\Gamma(I)](x) = \max\{I(y) \mid y \in \Gamma^{-1}(x)\}$
- $[\epsilon_\Gamma(I)](x) = \min\{I(y) \mid y \in \Gamma(x)\}$

Exercise

- Write an algorithm whose data are a graph (E, Γ) and a grayscale image I on E and whose result is the image $I' = \delta_\Gamma(I')$